**Papers written by**

**Australian Maths**

**Software**

**SEMESTER ONE**

**MATHEMATICS SPECIALIST REVISION 1**

**UNIT 3**

**2016**

**SOLUTIONS**

Mathematics Specialist Unit 3, 2016, Semester One Solutions

**Section One**

1. (10 marks)

(a)

1 1 1 2

⎡ ⎤

⎢ ⎥ −

1 1 1 6

1 0 1 4

⎣ ⎦

1 1 1 2

⎡ ⎤

⎢ ⎥ − − − 0 2 0 4

*R R R R*

✔ ✔

1 2 1 3

⎣ − ⎦

0 1 0 2

1 1 1 2

⎡ ⎤

⎢ ⎥ −

1 2 0 4

⎣ ⎦ −

0 0 0 0 2

*R R*

✔

2 3

0 0

=

∴ There are an infinite number of solutions.

None of the planes are parallel.

There are either two or three identical planes or the three planes intersect in a common line.

None of the planes have identical/equivalent equations so the three planes ✔

meet in a common line. (4)

(b) Two of the planes are parallel. Therefore there is no intersection. ✔

*x* + *y* + *z* = 2 and − *x* − *y* − *z* =1 ⇔*x* + *y* + *z* = −1 are parallel (2)

✔

(c)

1 1 1 6

⎡ ⎤

⎢ ⎥ −

2 1 1 1

⎣ − − − ⎦

3 1 1 2

1 1 1 2

⎡ ⎤

⎢ ⎥ − − 0 1 3 11 2 3 *R R R R*

✔ ✔

1 2 1 3

0 4 4 20

⎣ ⎦

1 1 1 2

⎡ ⎤

⎢ ⎥

0 1 3 11

⎣ ⎦ ÷

0 1 1 5 4

*R*

3

1 1 1 2

⎡ ⎤

⎢ ⎥

0 1 3 11

⎣ ⎦ −

0 0 2 6

*R R*

✔

2 3

*z*

=

3

*y y*

+ = → =

9 11 2

*x x*

+ + = → =

2 3 6 1

The point of intersection is (1, 2, 3) (4) ✔

2

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2. (6 marks)

(a) f (x) = sin ( x ) ✔✔ (2) (b) ( ) ✔✔ (2) 3 g x = x +1 (c) (2) ( )

x 1 p x e

− =

✔

( ) ( )

q x 1 ln x

= +

✔

3. (13 marks)

(a) 3 2

*z* − *z* − 4 = 0

( )

3 2

*P z z z*

= − −

4

Let

( )

*P*

2 8 4 4 0

= − − =

✔

∴ = −

*z z*

2 2

so isa factor

2

*z z*

+ +

2

)

3 2

*z z z*

− + −

0 4

*z* ***-***

2

( )

3 2

− −

*z z*

2

2

*z z*

+

0

( )

2

− −

*z z*

2

2 4

*z*

−

( )

− −

2 4

*z*

0

2

*z z z*

= + + =

2 2 0

✔

or

2

− ± −

*b b ac*

4

*za*

=

2

− ± −

1 1 8

2

= Δ = − = 7 7

*i*

2

− ±

1 7

*i*

*z*

=

2

− ±

1 7

*i*

∴ ~~=~~ =

*z z*

2

2or ✔✔

(4)

3

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(b) ( )

3

*z cis*

= π 8

✔✔

3

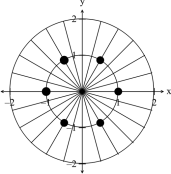
*z*

= −

8

✔

(c) (i)



(ii) 6

*z* =1

( )

6

(3)

✔✔ -1 per error

(2)

*z cis n n R* = + π ∈ 0 2

1

( ( ))

*z cis n*

= π

2

6

⎛ π ⎞ = ⎜ ⎟

2

*n*

*z ~~cis~~*

✔

6

⎝ ⎠

⎛ π ⎞ = ⎜ ⎟

*n*

*z ~~cis~~*

3

⎝ ⎠

( )

*n z cis*

= = = 0 0 1

***,***

⎛ π ⎞ = ~~= = +~~ ⎜ ~~⎟~~

1 3

*i*

✔

*n ~~z cis~~*

13 2 2

***,***

⎝ ⎠

⎛ π ⎞ = ~~= = − +~~ ⎜ ~~⎟~~

2 1 3

*i*

*n ~~z cis~~*

23 2 2

***,***

✔

⎝ ⎠

( )

*n z cis*

= = π = − 3 1

***,***

⎛ π ⎞ = ~~− = − = −~~ ⎜ ~~⎟~~

1 3

*i*

✔

*n ~~z cis~~*

13 2 2

***,***

⎝ ⎠

-1 per error (4) ⎛ π ⎞ = ~~− = − = − −~~ ⎜ ~~⎟~~

2 1 3

*i*

*n ~~z cis~~*

23 2 2

***,***

⎝ ⎠

4

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4. (8 marks)

( )

*x*

+

3

(a) ( ~~)~~

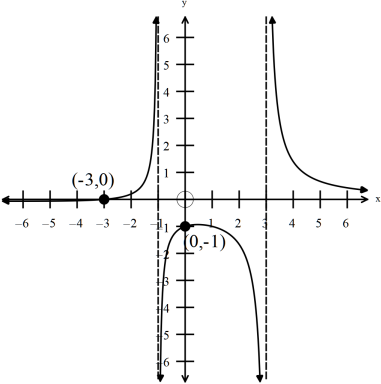
*f ~~x~~*

=+ −

( )( )

*x x*

1 3

✔✔

Vertical asymptotes at *x* = −1, *x* = 3

*x* intercept at *x* = −3

✔

*y* intercept at *y* = −1

✔

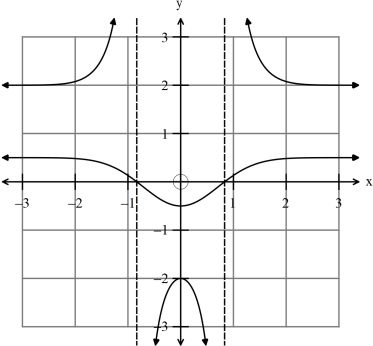
General shape – maximum turning point (w/0 cutting x axis); limits as ; *x*→±∞ limits about asymptotes -1/error (4) ✔

5

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(b)

5. (13 marks)

✔ 

✔ ✔

2 0 5− =−*x g x . e*

( )

✔

(4)

(a) . (3 3 + 2*i*)(2 3 − 2 2*i*) 2 18 2 6 6 6 4 = + − − *i i i*

✔✔

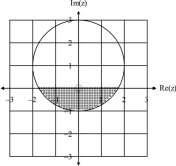
= −

22 4 6

*i*

✔

(b)

✔✔✔ -1/error

(3) (3)

6

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(c)

10

⎛ ⎛ π ⎞⎞ − ~~=~~ ⎜ ⎜− ⎟⎟

10

( ~~)~~

1 ~~2~~

*i ~~cis~~*

4

⎝ ⎝ ⎠⎠

⎛ π ⎞ = ~~−~~⎜ ⎟

10

10

( ~~)~~

24

*cis*

⎝ ⎠

⎛ π ⎞ = ~~−~~⎜ ⎟

5

5

22

*cis*

✔

⎝ ⎠

⎛ π ⎞ = ~~−~~⎜ ⎟ 322

*cis*

⎝ ⎠

( )

= −

32 0

*i*

10

( )

1 32

− = − *i i*

✔

(d) ( ) 3 4 *i Re Re i*

⎛ − ⎞

(2)

⎜ ⎟ = − − = −

1 2 1

⎝ + ⎠

1 2

*i*

✔ ✔ (2) (e) (i) ✔ (1) 2 ~~1 3~~ 1 3

*z ~~cis~~ i ~~z~~ i* ⎛ π ⎞ = ⎜ ~~⎟~~ = ~~− + ⇒= − −~~ 3 2 2 2 2

⎝ ⎠

2

*z ~~cis cis~~ i* ⎛ π ⎞ ⎛ π ⎞ = ⎜ ~~⎟~~ = ⎜ ~~⎟~~ = ~~− −~~

2 2 4 1 3

(ii) ✔ (1) 3 3 2 2

⎝ ⎠ ⎝ ⎠

1 1 1 4 1 3⎛ π ⎞ = ~~= =~~ ⎜− ⎟ = ~~− +~~⎛ ⎛ π ⎞⎞⎛ π ⎞ ⎝ ⎠

(ii) ✔ (1) 2 2

*cis i*

24 3 2 2

*zcis cis*

⎜ ⎟

⎜ ⎜ ⎟⎟⎝ ⎠

3 3

⎝ ⎝ ⎠⎠

**END OF SECTION ONE**

7

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**Section Two**

6. (6 marks)

(a) ***v*** (*t*) = (2*cos*(2*t*))***i***+ (−*sin*(*t*)) ***j***

∫

( ) (( ( )) ( ( )) )

***r i j***

*t cos t sin t dt*

= + −

2 2

( ) ( ( )) ( ( ))

***r i j c***

*t sin t cos t*

= + +

2

✔

( )

***r j***

π = −

*so*

( ( )) ( ( ))

− = π + π +

***j i j c***

*sin cos*

2

✔

− = − + ⇒ =

***j j c c 0***

( ) ( ( )) ( ( ))

∴ = +

***r i j***

*t sin t cos t*

2

( ) ( ( )) ( ( ))

***a i j***

*t sin t cos t*

= − + −

4 2

✔

(3)

⎛ π ⎞ ⎛ ⎛ π ⎞⎞ = ~~π +~~ ⎜ ⎟ = + = ⎜ ~~⎟ ⎜~~ ⎟

(b) ( ( ~~)~~) ✔ 3 3

***r*** *sin* ***i*** *cos* ***j i j 0***

3 0 0

2 2

⎝ ⎠ ⎝ ⎝ ⎠⎠

***v*** *cos* ***i*** *sin* ***j i j*** ⎛ π ⎞ ⎛ ⎛ π ⎞⎞ = ~~π +~~ ⎜− ⎟ = − + ⎜ ~~⎟ ⎜~~ ⎟

( ( ~~)~~) ✔ (2) 3 3

2 ~~3~~ 2

2 2

⎝ ⎠ ⎝ ⎝ ⎠⎠

(c) Show that 4***r***(*t*)***+ a*** (*t*) = 3*cos*(*t*) ***j***.

( )

( )

⎛ ⎞ ⎛− ⎞ = ⎜ ⎟ + ⎜ ⎟ − ⎝ ⎠ ⎝ ⎠

*sin t sin t*

2 4 2

( ) ( )

4 4

***r + a***

*t t*

( )

( )

*cos t cos t*

0

⎛ ⎞ = ⎜ ⎟

✔ (1)

( )

3

*cos t*

⎝ ⎠

( ( ))

=

3

*cos t*

***j***

7. (20 marks)

4

***r*** *cos* ***i*** *sin* ***j*** ⎛ ⎞ = + = ⎜ ⎟

(a) ( ) ( ( )) ( ( )) ✔

0 4 0 3 0

0

⎝ ⎠

⎛ ⎞ = + = ⎜ ⎟

−

( ) ( ( )) ( ( )) ✔ 4

+ + +

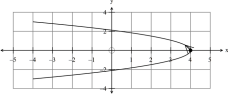
***r*** *cos* ***i*** *sin* ***j***

0 4 0 3 0

+

0

⎝ ⎠

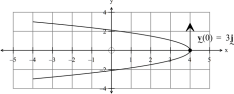
✔

(3)

8

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(b) ***v*** (*t*) = (−8*sin*(2*t*))***i*** + (3*cos*(*t*)) ***j*** ✔✔ ***v*** (0) = 3 ***j*** ✔

✔

(4)

(c) ***a*** (*t*) = (−16*cos*(2*t*))***i*** + (−3*sin*(*t*)) ***j*** ✔✔ (2) 16

⎛ ⎞ π

(d) ( ~~)~~ ✔

***a*** *t ~~= t~~*

⎜ ~~⎟~~ ⇒ ~~=~~ ⎝− ⎠

3 2

⎛ π ⎞ ⎛ ⎛ π ⎞⎞

(4 ( ~~)~~) 3

***r ~~i~~ j***

⎜ ~~⎟~~ = ~~π +~~ ⎜ ⎜ ⎟⎟

*cos ~~sin~~*

2 2

⎝ ⎠ ⎝ ⎝ ⎠⎠

⎛ π ⎞ = − + ⎜ ⎟

***r i j***

4 3

✔

2

⎝ ⎠

(2)

(e) ***a*** (*t*) = (−16*cos*(2*t*))***i*** + (−3*sin*(*t*)) ***j r***(*t*) = (4*cos*(2*t*))***i*** + (3*sin*(*t*)) ***j***.

***a*** (*t*) ≠ *k* ***r***(*t*) ✔

as −1×3 = −3 *but* −1× 4 ≠ −16 ✔ (2)

(f) If then ***a*** (*t*)*=* ***0 a*** (*t*) = (−16*cos*(2*t*))***i*** + (−3*sin*(*t*)) ***j*** =***0***✔

16 (2 ) 0 ( 3 ( )) 0 *x cos t* and *y sin t* = − = = − = π

2 0 2 *t k t*

= + π = π π

***, ,***

2

✔

π π

*k*

*t*

= ~~+~~

4 2

π π π π

3 5 7

*t*

=

***, ~~, ,~~***

✔

4 4 4 4

✔

The x and y coordinates are never zero at the same time so (4) ***a***(*t*)≠***0***

9

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(g) ***r***(*t*) = (4*cos*(2*t*))***i*** + (3*sin*(*t*)) ***j*** ✔

( ( )) ( ( )) *x cos t y sin t* = = 4 2 3 *y*

( ( ~~)~~) ( ~~)~~

2

*x ~~sin t sin t~~* = ~~− =~~

4 ~~1 2~~

✔

3

⎛ ⎞ = ⎜ − ⎟

2

2

*y*

✔

*x*

4 ~~1~~

9

⎝ ⎠

(3)

8. (10 marks)

(a) *A*(3***,*** 4***,*** 0) ***,*** *B*(4***,*** − 3***,*** 0) and *C*(0***,*** 0***,*** 5). ⎛ ⎞ ⎛ − ⎞

1 3

⎜ ⎟ ⎜ ⎟ = − = −

✔

***AB*** , ***AC***

7 4

There are alternative answers as they

⎜ ⎟ ⎜ ⎟

can also use ***BC*** as a direction vector

0 5

⎝ ⎠ ⎝ ⎠

and then any of the points A,B or C for

⎛ ⎞ ⎛ ⎞ ⎛ − ⎞

3 1 3

the equation.

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = + − + −

( )

***r*** *t s t*

4 7 4

✔

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

0 0 5

⎝ ⎠ ⎝ ⎠ ⎝ ⎠

(2)

(b) ✔✔ (2) 2 2 2 *x* + *y* + *z* = 25

⎛− ⎞ ⎛ ⎞

3 4

⎜ ⎟ ⎜ ⎟ = +

(c) (i) ( )

***r****ball t t*

5 3

✔

⎜ ⎟ ⎜ ⎟

− ⎝ ⎠ ⎝ ⎠

2 1

Hits the ground at 2 − *t* = 0 i.e. *t* = 2 seconds ✔ (2) 3 1

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ = +

(ii) ( )

✔

***r .*** *Paul t t*

4 3 5

⎜ ⎟ ⎜ ⎟

− ⎝ ⎠ ⎝ ⎠

2 1

3 1 5

⎛ ⎞ ⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = + =

✔

( )

***r .*** *Paul*

2 4 2 3 5 11

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ − ⎝ ⎠ ⎝ ⎠ ⎝ ⎠

2 1 0

⎛− ⎞ ⎛ ⎞ ⎛ ⎞

3 4 5

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ = + =

( )

***r****ball*

2 5 2 3 11

✔

⎜ ⎟ ⎜ ⎟ ⎜ ⎟

⎜ ⎟ ⎜ ⎟ ⎜ ⎟ − ⎝ ⎠ ⎝ ⎠ ⎝ ⎠

2 1 0

Both Paul and the ball are at the same point at ground level *Q*(5***,***11***,***0)

when , so Paul catches the ball. (3) *t* = 2

10

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1

⎛ ⎞

⎜ ~~⎟~~ × = × + + = (ii) (1) 2 3 5 2 1 12 25 1 7 55 ***. . .*** m

⎜ ⎟ −⎝ ⎠

✔

9. (6 marks) (a) (i)

1

⎛− ⎞

4

⎜ ⎟ = −

✔

***MN***

6

−⎝ ⎠

8

⎛ ⎞ ⎛− ⎞

2 4

⎜ ⎟ ⎜ ⎟ = + −

( ) (2) *r t t*

1 6

✔

⎜ ⎟ ⎜ ⎟

− ⎝ ⎠ ⎝ ⎠

3 8

✔

(ii) *x* = 2 − 4*t y* =1− 6*t z* = 3−8*t* 2 1 3

− − −

*x y z*

*t ~~t t~~*

= ~~= =~~

4 6 8

2 1 3

− − −

*x y z*

✔

*so*

= ~~=~~

4 6 8

(2)

⎛ ⎞ ⎛− ⎞ = −

1 1 1

*x t*

⎜ ⎟ ⎜ ⎟ = + ⇒=

(b) 1 1 ( )

*L t t y*

: ***r***

2 0 2

⎜ ⎟ ⎜ ⎟

− = − ⎝ ⎠ ⎝ ⎠

3 1 3

*z t*

⎛ ⎞ ⎛ ⎞ =

0 0 0

*x*

⎜ ⎟ ⎜ ⎟ = + ⇒=

2 2 ( )

*L s s y*

: ***r***

2 0 2

⎜ ⎟ ⎜ ⎟

= + ⎝ ⎠ ⎝ ⎠

2 2 2 2

*z s*

It can be seen that . If *y* = 2 *x* = 0 ⇒*t* =1 *i****.****e****.*** *z* = 2 ⇒(0***,***2***,***2)If then which does not contradict the and values and gives *z* = 2 *s* = 0 *x y*

(0***,***2***,***2).

✔ logic

Yes, the lines intersect, and do so at . (0***,***2***,***2)

✔

(2)

11

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10. (9 marks)

1 2

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ = = −

***AB AC***

2 4

,

✔

(a)

⎜ ⎟ ⎜ ⎟

− − ⎝ ⎠ ⎝ ⎠

4 6

( ) ( ) 12

*Area a b sin C and* ***a b a b*** *sin* Δ = × × × = θ

1

*Area*

= ×

***AB AC***

Δ

2

✔

⎛− ⎞

28

12

⎜ ⎟ = −

✔

28

− ⎝ ⎠

1784 4 64

= + +

2

2

*Area units*

=

14 6

***.***

Δ

✔

(4)

(b) (2) 7 ~~3~~

×

*AB* : *AC* ***AB*** : ***AC*** : :

= ~~= =~~ =

21 ~~56~~ 3 2 2

7 8

×

✔ ✔

(c) *P*(1***.***5***,***3***,***1) *Q*(2***,***0***,***0)

✔

0 5 1

***.***

⎛ ⎞ ⎛ ⎞

⎜ ⎟ ⎜ ⎟ = − = −

***PQ BC***

3 6

✔

⎜ ⎟ ⎜ ⎟

− − ⎝ ⎠ ⎝ ⎠

1 2

0 5

***.***

⎛ ⎞

⎜ ⎟ = −

✔

***BC***

2 3

−⎝ ⎠

1

∴ =

2

***PQ BC***

Therefore is parallel to (3) *PQ BC*.

12

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11. (13 marks)

3

⎛ ⎛ π ⎞⎞

3

( )

⎜ ⎟ − ⎜ ⎟

*cis i*

1

3

( ~~)~~ ( )

*xy*

4

⎝ ⎝ ⎠⎠ =

a

12

*zi*

( )

1 3

+

⎛ ⎛ π ⎞⎞⎛ ⎛ π ⎞⎞

3 3

3

( ~~)~~

⎜ ⎜ ~~⎟~~⎟⎜ ⎜− ⎟⎟

*cis ~~cis~~*

2

4 4

⎝ ⎝ ⎠⎠⎝ ⎝ ⎠⎠ =

12

⎛ ⎛ π ⎞⎞

23

*cis*

⎜ ⎜ ⎟⎟

⎝ ⎝ ⎠⎠

⎛ ⎛ π ⎞⎞⎛ ⎛ π ⎞⎞

3 3

2 ~~2~~

⎜ ⎜ ~~⎟~~⎟⎜ ⎜− ⎟⎟

*cis ~~cis~~*

4 4

⎝ ⎝ ⎠⎠⎝ ⎝ ⎠⎠

✔

=

⎛ ⎛ π ⎞⎞

2

*cis*

⎜ ⎜ ⎟⎟

6

⎝ ⎝ ⎠⎠

⎛ ⎛ π ⎞⎞

=34π

3

π

✔

24

⎜ ⎜ − ⎟⎟

*cis*

−6

⎝ ⎝ ⎠⎠

⎛ ⎛ π ⎞⎞ = ⎜ ⎜− ⎟⎟

26

*cis*

✔

⎝ ⎝ ⎠⎠

⎛ ⎞ = ⎜ − ⎟

3

*i*

22 2

⎝ ⎠

✔

= −

3

*i*

(4)

(b) ( ~~)~~ ✔ correct inequalities (3) 3

⎧ π π ⎫ ⎨ ≤ ~~≤ ∩ − ≤ ≤~~ ⎬

*z*: *z ~~arg z~~*

1 ~~2~~

4 4

⎩ ⎭

✔ ✔

(c) (i) Given show that 1 1 1 2 2 2 *z* = *x* + *iy* and *z* = *x* + *iy*1 1 2 *z z*2 =*z* ×*z .* ( ) ( )

*z z x iy x iy*

× = + × +

2 1 1 2 2

1

✔

( ) ( )

= − × −

*x iy x iy*

1 1 2 2

( )

2

= + + − −

*x x i y y i x y x y*

1 2 1 2 1 2 2 1

( )

= − − +

*x x y y i x y x y*

✔

1 2 1 2 1 2 2 1

( )( )

*z z x iy x iy*

= + +

2 1 1 2 2

1

2

= + + +

*x x i y y ix y ix y*

1 2 1 2 1 2 2 1

( )

= − + +

*x x y y i x y x y*

1 2 1 2 1 2 2 1

( )

= − − +

*x x y y i x y x y*

✔

1 2 1 2 1 2 2 1

= ×

*z z*

1 2

(3)

13

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(ii) *z* = *x* + *iy x* = *? y* = *?*

( ) ( )

*z i z i z i*

1 1 2 10 2

+ + − + = −

( )( ) ( )( )

*x iy i x iy i x iy i* + + + − − + + = − 1 1 2 2 10 2

✔

( ) ( ) ( ) ( ) *x y i x y x y i x y x iy i* − + + + − + − − + + =−

2 2 102

( ) ( ) *x y x y x i x y y x y i* − + − + + + − − + = −

2 2 10 2 ( ) ( )

4 2 2 10 2

*x y i y i*

− + = −

*Im : y*

2 2

= −

*y*

= −

1

*Re : x y*

4 2 10

− =

4 2 10

*x*

+ =

*x , y*

= = −

2 1

✔

✔

(3)

12. (6 marks)

(a) ***a*** (*t*) = −9***.***8 ***j***

∫

✔

( )

***v . j . j c***

*t dt t*

= − = − +

9 8 9 8

1

( ) ( ) ( )

0 0

***v i j i j c i j***0 20 60 20 60 10 10 3 10103= + = + ⇒=+

*cos sin*

1

( ) ( )

✔

***v i . j***

*t t*

= + −

10 10 3 9 8

( ) ( ) ( )

∫

2

***r i . j i . j c****t t dt t t t* = + − = + − +

10 10 3 9 8 10 10 3 4 9 2

0

⎛ ⎞ = ⎜ ⎟ ⇒ = ( )

***r c j***

01

2

⎝ ⎠

( ) ( )

2

✔

***r i . j***

*t t t t*

= + − +

10 10 3 4 9 1

✔

2

*If t t and h t t*

10 50 5 10 3 4 9 1 = = = − +

,

***.***

*At t h m* = = −

5 34 9

***, .***

This means the ball is not in flight for five seconds so Tom could not have

kicked the ball through the window.

✔

(4)

2 10 3 4 9 1

*h t . t*

= − +

(b)

*At t , h . m* = =

3 8 9

✔

The ball was still in flight so the deputy may have seen it. (1)

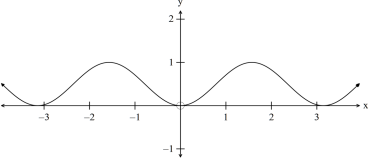
14

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13. (14 marks)

(a) (i) ( ( )) ( ( )) ( ( )) (1) 2 *y* = *g f x* = *g sin x* = *sin x* ✔

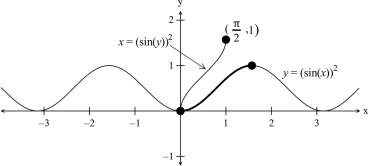
(ii)

✔✔✔ -1/error (3)

*a*π

(iii) ✔✔ (2) 2 =

(iv)

✔✔ 

✔

(2)

(b) (i) *h*( *x*) = *x*

( ) (1) 1 2 *h x x* for *x* 0 − = ≥

✔ *must have restricted domain*

(ii) (1) *h*( *x*) ≥ 0

✔

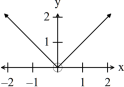
✔

(c) g(h (x)) = g( x ) = x ✔  Defined on [0,2π] ✔ (2)

15

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(d) (i)

✔ 

✔

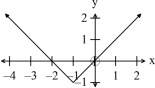
(1)

(ii) has no inverse because it is not a one to one function y = x

i.e. for every there is not a unique . x y ✔ (1)

14. (7 marks)

(a)

✔ 

✔

y = x +1 −1

so x < −2 or x > 0 ✔ (2)

(b) Solve 1+ *x* −1= *x* −1

*x*

⎧ = + − = ⎨⎩ − −

for x

≥ −

1

✔

*y x*

1 1

*x*

2

for x

< −

1

✔

⎧ −

*x*

1

for x

≥

1

*y x*

= − = ⎨⎩ −

11

✔

*x*

for x

<

1

✔

For x ≥1

x = x −1 No solution

✔

✔

For −1< x <1

x 1 x

= −

2x 1

=

1

x2

=

✔

✔

For x < −1

− − = − +

x 2 x 1

✔

− =

2 1

No solution

✔

1

∴ ~~=~~

x2

only

(5)

16

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15. (4 marks)

✔✔

2

x

✔

yx 1 x 3 =− + ( )( )

✔

✔

16. (5 marks)

✔ ✔

(i)

(2)

(1 i) 1 i 1 i

− = − = − 2

(1 i) 2i 2i

− = − = − 3

(1 i) 2 2i 2(1 i)

− = − − = − + 4

(1 i) 4 4

− = − = − 5

(1 i) 4 4i 4(1 i)

− = − + = − − 6

(1 i) 8i 4(2i)

− = = 7

(1 i) 8 8i 8(1 i)

− = + = + 8

(1 i) 16 16

− = = 9

(1 i) 16 16i 16(1 i)

− = − = − 10

(1 i) 32i 32i

− = − = − ✔✔

✔

(ii) Every fourth result seems to be connected.

✔

Starting with , then , the pattern seems to be a multiple of n =1 n = 5 (1−i). Starting with , then , the pattern seems to be a multiple of n = 2 n = 6 i . Starting with , then , the pattern seems to be a multiple of n = 3 n = 7 i . Starting with , then , the pattern seems to be a real multiple of 4. n = 4 n = 8 ✔ *for any one of these up to 3 marks*

The coefficients are a pattern of powers of 2. ✔

A lot more analysis is needed, but this is sufficient for 3 marks. (3)

**END OF SECTION TWO**

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